

# MagneTrack: Magnetic Field Separation Method for Continuous and Simultaneous 1-DOF Tracking of Two-magnets

ANONYMOUS AUTHOR(S)\*

## ACM Reference Format:

Anonymous Author(s). 2021. MagneTrack: Magnetic Field Separation Method for Continuous and Simultaneous 1-DOF Tracking of Two-magnets. 1, 1 (December 2021), 2 pages. <https://doi.org/10.1145/nnnnnnn.nnnnnnn>

## 1 APPENDIX

This appendix shows the derivation process of a magnetic field separation which is omitted in 3.3 separation process.

### 1.1 Separation Process

We obtained the magnetic field difference generated from each magnet  $Y_\phi$  as the observation data, as follows:

$$Y_\phi = \sum_i M_{i,\phi} + \epsilon_\phi. \quad (1)$$

As the given magnetic field value is real, we separated the observation data and the basis vector into positive and negative components applied to NMF:

$$X_{i,\phi}^{(+)} = \max(0, X_{i,\phi}) \quad X_{i,\phi}^{(-)} = \min(0, X_{i,\phi}) \quad (2)$$

$$Y_\phi^{(+)} = \max(0, Y_\phi) \quad Y_\phi^{(-)} = \min(0, Y_\phi) \quad (3)$$

Therefore, when the label of a sign is defined as  $s \in \{(+), (-)\}$ , the magnetic field difference model and the observation data can be transcribed to

$$M_{i,\phi}^s = H_i X_{i,\phi}^s \quad (4)$$

$$Y_\phi^s = \sum_i M_{i,\phi}^s + \epsilon_\phi^s, \quad (5)$$

respectively. In theory, the error in the observation data  $\epsilon_\phi^s$  is zero. The minimization problem of the error is addressed by minimizing

$$\mathcal{J}(Y, M) \equiv \sum_\phi D(Y_\phi, M_\phi) + 2\lambda_L \sum_i |H_i| \quad (6)$$

with respect to  $H_i$ . Here, the second term is the regularization parameter that prevents  $H_i$  from being overfitted when an empirical value is assigned to  $\lambda_L$ . The first term indicates  $\beta$  divergence. Even though the  $\beta$  divergence is a convex function whose shape can be transformed variedly, we adopted Euclidean distance in the case of  $\beta = 2$ :

$$D(Y_\phi, M_\phi) = \sum_s (Y_\phi^s - \sum_i M_{i,\phi}^s)^2. \quad (7)$$

---

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from [permissions@acm.org](mailto:permissions@acm.org).

© 2021 Association for Computing Machinery.

Manuscript submitted to ACM

Manuscript submitted to ACM

Based on the auxiliary function method, the auxiliary function of the cost function given by Eq. (6) is derived using the following pair of inequality expressions.

$$(Y_\phi^s - \sum_i M_{i,\phi}^s)^2 \leq \sum_i \frac{(\hat{Y}_{i,\phi}^s - H_i X_{i,\phi}^s)^2}{\beta_{i,\phi}^s} \quad (8)$$

$$|H_i| \leq \frac{|H_i|}{2} H_i^2 + |\hat{H}_i| - \frac{|\hat{H}_i|}{2} \quad (9)$$

Note that Eq. (8) holds under the conditions that  $\sum_i \hat{Y}_{i,\phi}^s = Y_\phi^s$ , and  $\beta_{i,\phi}^s$  is an arbitrary value satisfying  $0 \leq \beta_{i,\phi}^s \leq 1$  and  $\sum_i \beta_{i,\phi}^s = 1$ . Thus, assuming that

$$J^+(Y, M) \leq \sum_{s,i,\phi} \frac{(\hat{Y}_{i,\phi}^s - H_i X_{i,\phi}^s)^2}{\beta_{i,\phi}^s} + \lambda_L (|\hat{H}_i|^{-1} H_i^2 + |\hat{H}_i|), \quad (10)$$

then  $\mathfrak{J}(Y, M) \leq J^+(Y, M)$ , so that in the case of

$$\hat{Y}_{i,\phi}^s = H_i X_{i,\phi}^s + \beta_{i,\phi}^s (Y_\phi^s - M_{i,\phi}^s) \quad (11)$$

$$\hat{H}_i = H_i \quad (12)$$

$J^+(Y, M)$  fulfills the definition of auxiliary function because  $\mathfrak{J}(Y, M) = J^+(Y, M)$ . Note that the solution can be obtained effectively by assigning

$$\beta_{i,\phi}^s = \frac{H_i X_{i,\phi}^s}{\sum_n H_n X_{n,\phi}^s} \quad (13)$$

under the above stated conditions. The update function can be obtained using  $\partial J^+ / \partial H_i = 0$  as follows:

$$H_i = \frac{\sum_{s,\phi} (\hat{Y}_{i,\phi}^s X_{i,\phi}^s / \beta_{i,\phi}^s)}{\sum_{s,\phi} ((X_{i,\phi}^s)^2 / \beta_{i,\phi}^s) + \lambda_L |\hat{H}_i|^{-1}} \quad (14)$$

By iteratively updating Eq. (4), (11), (12), (13) and (14), the solution will converge to a stationary point. In our study, we set zero to  $\lambda_L$ .