# MagneTrack: Magnetic Field Separation Method for Continuous and Simultaneous 1-DOF Tracking of Two-magnets

ANONYMOUS AUTHOR(S)*

**ACM Reference Format:**

Anonymous Author(s). 2021. MagneTrack: Magnetic Field Separation Method for Continuous and Simultaneous 1-DOF Tracking of Two-magnets. 1, 1 (December 2021), 2 pages. https://doi.org/10.1145/nnnnnnn.nnnnnnn

## 1 APPENDIX

This appendix shows the derivation process of a magnetic field separation which is omitted in 3.3 separation process.

### 1.1 Separation Process

We obtained the magnetic field difference generated from each magnet $Y_\phi$ as the observation data, as follows:

$$Y_\phi = \sum_i M_{i,\phi} + \epsilon_\phi. \tag{1}$$

As the given magnetic field value is real, we separated the observation data and the basis vector into positive and negative components applied to NMF:

$$X^{(+)}_{i,\phi} = \max(0, X_{i,\phi}) \quad X^{(-)}_{i,\phi} = \min(0, X_{i,\phi}) \tag{2}$$

$$Y^{(+)}_\phi = \max(0, Y_\phi) \quad Y^{(-)}_\phi = \min(0, Y_\phi) \tag{3}$$

Therefore, when the label of a sign is defined as $s \in \{(+), (-)\}$, the magnetic field difference model and the observation data can be transcribed to

$$M^s_{i,\phi} = H_i X^s_{i,\phi} \tag{4}$$

$$Y^s_\phi = \sum_i M^s_{i,\phi} + \epsilon^s_\phi, \tag{5}$$

respectively. In theory, the error in the observation data $\epsilon^s_\phi$ is zero. The minimization problem of the error is addressed by minimizing

$$\mathcal{J}(Y,M) \equiv \sum_\phi D(Y_\phi, M_\phi) + 2\lambda_\mathrm{L} \sum_i |H_i| \tag{6}$$

with respect to $H_i$. Here, the second term is the regularization parameter that prevents $H_i$ from being overfitted when an empirical value is assigned to $\lambda_\mathrm{L}$. The first term indicates $\beta$ divergence. Even though the $\beta$ divergence is a convex function whose shape can be transformed variedly, we adopted Euclidean distance in the case of $\beta = 2$:

$$D(Y_\phi, M_\phi) = \sum_s (Y^s_\phi - \sum_i M^s_{i,\phi})^2. \tag{7}$$


Manuscript submitted to ACM

Based on the auxiliary function method, the auxiliary function of the cost function given by Eq. (6) is derived using the following pair of inequality expressions.

$$(Y^s_\phi - \sum_i M^s_{i,\phi})^2 \leq \sum_i \frac{(\hat{Y}^s_{i,\phi} - H_i X^s_{i,\phi})^2}{\beta^s_{i,\phi}} \tag{8}$$

$$|H_i| \leq \frac{|H_i|}{2} H_i^2 + |\hat{H}_i| - \frac{|\hat{H}_i|}{2} \tag{9}$$

Note that Eq. (8) holds under the conditions that $\sum_i \hat{Y}^s_{i,\phi} = Y^s_\phi$, and $\beta^s_{i,\phi}$ is an arbitrary value satisfying $0 \leq \beta^s_{i,\phi} \leq 1$ and $\sum_i \beta^s_{i,\phi} = 1$. Thus, assuming that

$$J^+(Y,M) \leq \sum_{s,i,\phi} \frac{(\hat{Y}^s_{i,\phi} - H_i X^s_{i,\phi})^2}{\beta^s_{i,\phi}} + \lambda_{\mathrm{L}}(|\hat{H}_i|^{-1} H_i^2 + |\hat{H}_i|), \tag{10}$$

then $\mathcal{J}(Y, M) \leq J^+(Y, M)$, so that in the case of

$$\hat{Y}^s_{i,\phi} = H_i X^s_{i,\phi} + \beta^s_{i,\phi}(Y^s_\phi - M^s_{i,\phi}) \tag{11}$$

$$\hat{H}_i = H_i \tag{12}$$

$J^+(Y, M)$ fulfills the definition of auxiliary function because $\mathcal{J}(Y, M) = J^+(Y, M)$. Note that the solution can be obtained effectively by assigning

$$\beta^s_{i,\phi} = \frac{H_i X^s_{i,\phi}}{\sum_n H_n X^s_{n,\phi}} \tag{13}$$

under the above stated conditions. The update function can be obtained using $\partial J^+ / \partial H_i = 0$ as follows:

$$H_i = \frac{\sum_{s,\phi}(\hat{Y}^s_{i,\phi} X^s_{i,\phi} / \beta^s_{i,\phi})}{\sum_{s,\phi}((X^s_{i,\phi})^2 / \beta^s_{i,\phi}) + \lambda_{\mathrm{L}} |\hat{H}_i|^{-1}} \tag{14}$$

By iteratively updating Eq. (4), (11), (12), (13) and (14), the solution will converge to a stationary point. In our study, we set zero to $\lambda_{\mathrm{L}}$.